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Periodic Positive Solution of a Predator-Prey System with Stage-Structure for Prey and Time Delay

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Abstract: A Predator-Prey system with time delay is considered. There are, immature and mature, two stage for prey species in the system. By using the continuation theorem of Gaines and Mawhin's coincidence degree theory, a sufficient condition is derived for the existence periodic positive solution.

Key words: coincidence degree; Predator-Prey system; periodic positive solution; stage structure
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一类食饵具有阶段结构的时滞 Predator-Prey 系统的周期解

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摘　要:研究了一类时滞 Predator-Prey 系统,其中 Prey 种群是具有两个生命阶段的种群,即幼年阶段和成年阶段。Predator 种群只能捕食 Prey 幼年种群。通过应用 Gaines 和 Mawhin 重合度理论的连续函数定理,给出了系统正周期解存在的充分条件。

关键词:重合度;Predator-Prey 系统;正周期解;阶段结构
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1 Introduction

The dynamics of population with delays is useful for the control of the population of mankind, animals and environment. One of the famous models for dynamics of population is predator-prey system. There is a large volume of literature relevant to the theory of the predator-prey system [1-4]. But the stage structure

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of species has been ignored in those articles. In this natural world, there are many species whose individual members have a life history that take them through stage structure, immature and mature. In particular, we have in mind mammalian populations and some amphibious animals, which exhibit these two stages. For example, Chinese fire-bellied newt, which is unable to prey the nature rana chensinensis, can only prey on the immature one. In recent years, permanence of predator-prey system with stage structure was discussed. Cui and Song proved a sufficient and necessary condition to guarantee the permanence of a predator-prey system with stage structure[5]. The effect of delay on the population at positive equilibrium and the optimal harvesting of the mature prey population were considered by Song and Chen[6]. Global asymptotical stability of a predator-prey system with stage structure for prey were studied by Zhang[7].

Our purpose in this paper is, by using the continuation theorem which was proposed in[8], to establish the existence of at least one positive-periodic solution of a predator-prey models with stage structure and delay

$$\begin{cases} \dot{x}_1 = a(t)x_2 - b(t)x_1 - d(t)x_1^2 - p(t)x_1\int_{-\tau}^{0} k_1(s)y(t+s)\mathrm{d}s \\ \dot{x}_2 = c(t)x_1 - f(t)x_2^2 \\ \dot{y} = y[-g(t) + h(t)x_1 - r(t)y - q(t)\int_{-\tau}^{0} k_2(s)y(t+s)\mathrm{d}s] \end{cases} \tag{1}$$

where x_1 and x_2 denote the density of immature and mature population A respectively, and y is the density of predator B that preys on x_1. The coefficients in (1) are all ω-periodic and continuous for $t\geqslant 0$, where $a(t)$, $b(t)$, $c(t)$, $d(t)$, $f(t)$, $g(t)$, $h(t)$ and $r(t)$ are positive, and $p(t)$, $q(t)$ are nonnegative. Here $K_i(s)(i=1,2)$ defined on $[-\tau,0](\tau\geqslant 0)$ are nonnegative and integrable satisfying $\int_{-\tau}^{0} K_i(s)\mathrm{d}s = 1(i=1,2)$.

In what follow, we use the following notation:

$$\bar{f} = \frac{1}{\omega}\int_0^{\omega} f(t)\mathrm{d}t,\ f^l = \min_{t\in[0,\omega]} |f(t)|,$$
$$f^m = \max_{t\in[0,\omega]} |f(t)|$$

where f has a periodic continuous function with periodic $\omega > 0$.

Main Theorem We assume the following:

$$\text{(H)} \qquad \frac{1}{r^m}\left\{\left[\left(\frac{f}{c}\right)^m\left(\left(\frac{d}{a}\right)^m\right)^2\right]^{-\frac{1}{3}} - q^m\left(\left(\frac{h}{r}\right)^m\left[\left(\frac{d}{a}\right)^l\left(\left(\frac{f}{c}\right)^l\right)^2\right]^{-\frac{1}{3}}\right)\right\} \geqslant g^l.$$

Then system (1) has at least one positive ω-periodic solution.

2 Proof of the main theorem

Before stating the main result one first states some notations. Let X and Y be real Banach space, L: $\mathrm{Dom}L \subset X \to Y$ a Fredholm mapping of index zero and $P: X\to$, $Q: Y\to Y$ continuous projections such that $\mathrm{Im}P = \mathrm{Ker}L$, $\mathrm{Ker}Q = \mathrm{Im}L$.

Introduction, for the sake of convenience, Mawhin's continuation theorem[7] as follows.

Lemma 1 Let L a Fredholm mapping of index zero. Assume that $N: \bar{\Omega} \to X$ is L-compact on $\bar{\Omega}$ with Ω open bounded in X. Furthermore assumption:

a) for each $\lambda \in (0,1)$, $x \in \partial\Omega \cap \mathrm{Dom}L$, $Lx \neq \lambda Nx$;

b) for each $x \in \partial\Omega \cap \mathrm{Ker}L$, $QNx \neq 0$;

c) $\deg\{QNx, \Omega \cap \mathrm{Ker}L, 0\} \neq 0$.

Then the operator equation $Lx = Nx$ has at least one solution in $\bar{\Omega}$.

Consider the system

$$\begin{cases}\dot{u}_1 = a(t)e^{u_2(t)-u_1(t)} - b(t) - d(t)e^{u_1(t)} - p(t)\int_{-\tau}^{0} k_1(s)e^{u_3(t+s)}ds, \\ \dot{u}_2 = c(t)e^{u_1(t)-u_2(t)} - f(t)e^{u_2(t)}, \\ \dot{u}_3 = -g(t) + h(t)e^{u_1(t)} - r(t)e^{u_3(t)} - q(t)\int_{-\tau}^{0} k_2(s)e^{u_3(t+s)}ds \end{cases} \tag{2}$$

It is easy to see that if system(2) is an ω-periodic solution $(u_1(t), u_2(t), u_3(t))$, then $(e^{u_1(t)}, e^{u_2(t)}, e^{u_3(t)})$ is a positive ω-periodic solution of system (1). Therefore, to have at least one positive ω-periodic solution for (1), it is sufficient that (2) has at least one ω-periodic solution.

Denote

$$X = \{(u_1(t), u_2(t), u_3(t))^T \in C(R, R^3) : u_i(t+w) = u_i(t), i = 1,2,3\}$$

and

$$\|(u_1(t), u_2(t), u_3(t))^T\| = \max_{t\in[0,\omega]} |u_1(t)| + \max_{t\in[0,\omega]} |u_2(t)| + \max_{t\in[0,\omega]} |u_3(t)|$$

with this norm. Thus X is a Banach space.

Let

$$N\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix} a(t)e^{u_2(t)-u_1(t)} - b(t) - d(t)e^{u_1(t)} - p(t)\int_{-\tau}^{0} k_1(s)e^{u_3(t+s)}ds \\ c(t)e^{u_1(t)-u_2(t)} - f(t)e^{u_2(t)} \\ -g(t) + h(t)e^{u_1(t)} - r(t)e^{u_3(1)} - q(t)\int_{-\tau}^{0} k_2(s)e^{u_3(t+s)}ds \end{bmatrix}$$

$$L\begin{bmatrix}u_1(t)\\u_2(t)\\u_3(t)\end{bmatrix} = \begin{bmatrix}\dot{u}_1(t)\\\dot{u}_2(t)\\\dot{u}_3(t)\end{bmatrix},\ p\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = Q\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix}\frac{1}{\omega}\int_0^{\omega} u_1(t)dt \\ \frac{1}{\omega}\int_0^{\omega} u_2(t)dt \\ \frac{1}{\omega}\int_0^{\omega} u_3(t)dt\end{bmatrix}, \text{where } \begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} \in X.$$

We obtain $\mathrm{Ker}L = R^3$, $\mathrm{Im}L$ is closed in X and L is a Fredholm mapping of index zero. By a simple computation we find that the inverse K_p of L has the form

$$K_p : \mathrm{Im}L \to \mathrm{Ker}P \cap \mathrm{Dom}L, \text{ and } K_p(z) = \int_0^t z(s)ds - \frac{1}{\omega}\int_0^{\omega}\int_0^t z(s)dsdt.$$

Therefore

$$QN\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix} \frac{1}{\omega}\int_0^{\omega}\left[a(t)e^{u_2(t)-u_1(t)} - b(t) - d(t)e^{u_1(t)} - p(t)\int_{-\tau}^{0} k_1(s)e^{u_3(t+s)}ds\right]dt \\ \frac{1}{\omega}\int_0^{\omega}\left[c(t)e^{u_1(t)-u_2(t)} - f(t)e^{u_2(t)}\right]dt \\ \frac{1}{\omega}\int_0^{\omega}\left[-g(t) + h(t)e^{u_1(t)} - r(t)e^{u_3(t)} - q(t)\int_{-\tau}^{0} k_2(s)e^{u_3(t+s)}ds\right]dt \end{bmatrix},$$

$$K_p(I-Q)N\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix} \int_0^t\left[a(z)e^{u_2(z)-u_1(z)} - b(z) - d(z)e^{u_1(z)} - p(z)\int_{-\tau}^{0} k_1(s)e^{u_3(z+s)}ds\right]dz \\ \int_0^t\left[c(z)e^{u_1(z)-u_1(z)} - f(z)e^{u_2(z)}\right]dz \\ \int_0^t\left[-g(z) + h(z)e^{u_1(z)} - r(z)e^{u_3(z)} - q(z)\int_{-\tau}^{0} k_2(s)e^{u_3(z+s)}ds\right]dz \end{bmatrix}$$

$$
-\begin{bmatrix}
\frac{1}{\omega}\int_0^\omega\int_0^t\left[a(z)e^{u_2(z)-u_1(z)}-b(z)-d(z)e^{u_1(z)}-p(z)\int_{-\tau}^0 k_1(s)e^{u_3(z+s)}ds\right]dzdt \\
\frac{1}{\omega}\int_0^\omega\int_0^t[c(t)e^{u_1(t)-u_2(t)}-f(t)e^{u_2(t)}]dzdt \\
\frac{1}{\omega}\int_0^\omega\int_0^t\left[-g(t)+h(t)e^{u_1(t)}-r(t)e^{u_3(t)}-q(t)\int_{-\tau}^0 k_2(s)e^{u_3(t+s)}ds\right]dzdt
\end{bmatrix}
+\begin{bmatrix}
(\frac{1}{2}-\frac{1}{\omega})\int_0^\omega\left[a(z)e^{u_2(z)-u_1(z)}-b(z)-d(z)e^{u_1(z)}-p(z)\int_{-\tau}^0 k_1(s)e^{u_3(z+s)}ds\right]dt \\
(\frac{1}{2}-\frac{1}{\omega})\int_0^\omega[c(z)e^{u_1(z)-u_2(z)}-f(z)e^{u_2(z)}]dt \\
(\frac{1}{2}-\frac{1}{\omega})\int_0^\omega\left[-g(z)+h(z)e^{u_1(z)}-r(z)e^{u_3(z)}-q(z)\int_{-\tau}^0 k_2(s)e^{u_3(z+s)}ds\right]dt
\end{bmatrix}.
$$

Thus QN and $K_p(I-Q)N$ are continuous. By using the Arzela-Ascoli theorem, one can see the $QN(\bar{\Omega})$ and $K_p(I-Q)N(\bar{\Omega})$ are relatively compact for any open bounded set $\Omega\subset X$. Thus, we have that N is L-compact on $\bar{\Omega}$. Corresponding to system (2), we have

$$
\begin{cases}
\dot{u}=\lambda\left[a(t)e^{u_2(t)-u_1(t)}-b(t)-d(t)e^{u_1(t)}-p(t)\int_{-\tau}^0 k_1(s)e^{u_3(t+s)}ds\right], \\
\dot{u}=\lambda[c(t)e^{u_1(t)-u_2(t)}-f(t)e^{u_2(t)}], \\
\dot{u}=\lambda\left[-g(t)+h(t)e^{u_1(t)}-r(t)e^{u_3(t)}-q(t)\int_{-\tau}^0 k_2(s)e^{u_3(t+s)}ds\right]
\end{cases}
\tag{3}
$$

Assume that $(u_1(t), u_2(t), u_3(t))^T\in X$ is a solution of (3) some $\lambda\in(0,1)$. By integrating (3) over the interval $[0,\omega]$,

$$\int_0^\omega\left[a(t)e^{u_2(t)-u_1(t)}-b(t)-d(t)e^{u_1(t)}-p(t)\int_{-\tau}^0 k_1(s)e^{u_3(t+s)}ds\right]dt=0, \tag{4}$$

$$\int_0^\omega[c(t)e^{u_1(t)-u_2(t)}-f(t)e^{u_2(t)}]dt=0, \tag{5}$$

and

$$\int_0^\omega\left[-g(t)+h(t)e^{u_1(t)}-r(t)e^{u_3(t)}-q(t)\int_{-\tau}^0 k_2(s)e^{u_3(t+s)}dx\right]dt=0. \tag{6}$$

From (4),(5),(6), we claim

$$\int_0^\omega|\dot{u}_1(t)|dt\leqslant 2\int_0^\omega\left[b(t)+d(t)e^{u_1(t)}+p(t)\int_{-\tau}^0 k_1(s)e^{u_3(t+s)}ds\right]dt$$

$$\int_0^\omega|\dot{u}_2(t)|dt<2\int_0^\omega f(t)e^{u_2(t)}dt,$$

$$\int_0^\omega|\dot{u}_3(t)|dt<2\int_0^\omega h(t)e^{u_1(t)}dt.$$

Hence, there exist $\xi_i\in[0,\omega]$ $(i=1, 2, 3)$ such that

$$u_i(\xi_i)=\max_{t\in[0,\omega]}u_i(t),\ i=1,2,3.$$

It follows that

$$a(t_1)e^{u_2(\xi_1)-u_1(\xi_1)}-b(\xi_1)-d(\xi_1)e^{u_1(\xi_1)}-p(\xi_1)\int_{-\tau}^0 k_1(s)e^{u_3(\xi_1+s)}ds=0, \tag{7}$$

$$c(\xi_2)e^{u_1(\xi_2)-u_2(\xi_2)}-f(\xi_2)e^{u_2(\xi_2)}=0, \tag{8}$$

and

$$-g(\xi_3)+h(\xi_3)e^{u_1(\xi_3)}-r(\xi_3)e^{u_3(\xi_3)}-q(\xi_3)\int_{-\tau}^0 k_2(s)e^{u_3(\xi_3+s)}ds=0 \tag{9}$$

which implies

$$c(\xi_2)e^{u_1(\xi_1)-u_2(\xi_2)} \geqslant f(\xi_2)e^{u_2(\xi_2)} \text{ and } a(\xi_1)e^{u_2(\xi_2)-u_1(\xi_1)} \geqslant d(\xi_1)e^{u_1(\xi_i)}.$$

Then, we have

$$e^{u_1(\xi_1)} \geqslant \frac{f(\xi_2)}{c(\xi_2)}e^{2u_2(\xi_2)} \tag{10}$$

and

$$e^{u_2(\xi_2)} \geqslant \frac{d(\xi_1)}{a(\xi_1)}e^{2u_1(\xi_1)} \tag{11}$$

From (10),(11),we get

$$e^{u_1(\xi_1)} \leqslant \left\{(\frac{f}{c})^l\left[(\frac{d}{a})^l\right]^2\right\}^{-\frac{1}{3}} = A_1 \tag{12}$$

and

$$e^{u_2(\xi_2)} \leqslant \left\{(\frac{d}{a})^l\left[(\frac{f}{c})^l\right]^2\right\}^{-\frac{1}{3}} = A_2 \tag{13}$$

Combining (6) with (11), we obtain

$$e^{u_3(\xi_3)} \leqslant (\frac{h}{r})^m e^{u_1(\xi_1)} \leqslant (\frac{h}{r})^m\left\{(\frac{f}{c})^l\left[(\frac{d}{a})^l\right]^2\right\}^{-\frac{1}{3}} = A_3 \tag{14}$$

Choose $\eta_i \in [0,\omega]$, $i = 1,2,3$ such that

$$u_i(\eta_i) = \min_{t\in[0,\omega]} u_i(t),\ i = 1,2,3.$$

Then

$$c(\eta_2)e^{u_1(\eta_1)-u_2(\eta_2)} \leqslant f(\eta_2)e^{u_2(\eta_2)} \quad \text{and} \quad a(\eta_1)e^{u_2(\eta_2)-u_1(\eta_1)} \leqslant d(\eta_1)e^{u_1(\eta_1)}$$

That is

$$e^{u_1(\eta_1)} \leqslant (\frac{f}{c})^m e^{2u_2(\eta_2)} \text{ and } e^{u_2(\eta_1)} \leqslant (\frac{d}{a})^m e^{2u_1(\eta_1)}. \tag{15}$$

According to (15) it follows that

$$e^{u_1(\eta_1)} > \left\{(\frac{f}{c})^m\left[(\frac{d}{a})^m\right]^2\right\}^{-\frac{1}{3}} = B_1, \tag{16}$$

and

$$e^{u_2(\eta_2)} > \left\{(\frac{d}{a})\left[(\frac{f}{c})^m\right]^2\right\}^{-\frac{1}{3}} = B_2. \tag{17}$$

By the condition (H) in the Main Theorem, we have

$$-g(\eta_3)+h(\eta_3)e^{u_1(\eta_3)}-r(\eta_3)e^{u_3(\eta_3)}-q(\eta_3)\int_{-\tau}^{0}k_2(s)e^{u_3(\eta_3+s)}ds = 0.$$

Notice

$$-g(\eta_3)+h(\eta_3)e^{u_1(\xi_1)}-r(\eta_3)e^{u_3(\eta_3)}-q(\eta_3)e^{u_3(\eta_3)} < 0$$

By (14), we have

$$e^{u_3(\eta_3)} \geqslant \frac{1}{r^m}\left\{\left[(\frac{f}{c})^m((\frac{d}{a})^m)^2\right]^{-\frac{1}{3}} - \left\{g^m + q^m((\frac{h}{r})^m\left[(\frac{d}{a})^l((\frac{f}{c})^l)^2\right]^{-\frac{1}{3}})\right\}\right\} = B_3 \tag{18}$$

By (12),(13),(14), it follows that there exist three constants d_1, d_2 and d_3 such that

$$\int_0^\omega |\dot{u}_1(t)|\,dt \leqslant 2\int_0^\omega\left[b(t)+d(t)e^{u_1(t)}+p(t)\int_{-\tau}^{0}k_1(s)e^{u_3(t+s)}ds\right]dt < 2\omega(\bar{b}+\bar{d}A_1+\bar{p}A_3) \overset{\text{def}}{=} d_1$$

$$\int_0^\omega |\dot{u}_2(t)|\,dt \leqslant 2\int_0^\omega f(t)e^{u_2(t)}dt < 2\omega\bar{f}A_2 \overset{\text{def}}{=} d_2,$$

$$\int_0^\omega |\dot{u}_3(t)|\,dt \leqslant 2\int_0^\omega h(t)e^{u_1(t)}dt < 2\omega\bar{h}A_2 \overset{\text{def}}{=} d_3.$$

From this, it follows that there exist three positive constants ρ_1, ρ_2 and ρ_3 such that

$$u_1(t_1) \geqslant -\rho_1,\ u_2(t_2) \geqslant -\rho_2,\ \text{and}\ u_3(t_3) \leqslant -\rho_3 \tag{19}$$

For all $t \in [0,\omega]$, we have

$$u_1(t) = u_1(t_1) - \int_1^t \dot{u}_1(s)\mathrm{d}s,\ u_2(t) = u_2(t_2) - \int_2^t \dot{u}_2(s)\mathrm{d}s,$$

and

$$u_3(t) = u_3(t_3) - \int_3^t \dot{u}_3(s)\mathrm{d}s$$

which imply

$$u_i(t) > -\rho_i - \int_0^\omega |\ u_1(s)\ |\ \mathrm{d}s > -\rho_i - d_i (i = 1,2,3).$$

Then

$$|\ u_i(t)\ | \leqslant \max\{\ln A_i, \rho_i + d_i\} \overset{\text{def}}{=} R_i (i = 1,2,3),$$

here $R_i (i = 1,2,3)$ are independent of λ.

Denote $M = R_1 + R_2 + R_3 + R_0$. Here R_0 is taken sufficiently large such that each solution $(\alpha,\ \beta,\ \gamma)$ of the following system:

$$\begin{cases} \bar{a}e^{\beta-\alpha} - \bar{b} - \bar{b} - \bar{p}e^{\gamma} = 0 \\ \bar{c}e^{\alpha-\beta} - \bar{f}e^{\beta} = 0 \\ -\bar{g} + \bar{h}e^{\alpha} - (\bar{r} + \bar{q})e^{\gamma} = 0 \end{cases} \tag{20}$$

satisfies $\|(\alpha,\beta,\gamma)\| = |\alpha| + |\beta| + |\gamma| < M$, provided that system (20) has at least one solution. Now we take

$$\Omega = \{(u_1(t),\ u_2(t),\ u^3(t))^{\mathrm{T}} \in X : \|(y_1, y_2, y_3)^{\mathrm{T}}\| < M\}.$$

This satisfies condition (a) of Lemma 1. So $(y_1, y_2, y_3)^{\mathrm{T}} \in \partial\Omega \cap \mathrm{Ker}L = \partial\Omega \cap R^3$, and $(y_1, y_2, y_3)^{\mathrm{T}}$ is a constant vector in R^3 with $|y_1| + |y_2| + |y_3| = M$. If system (20) has a solution or a number of solutions, then

$$QN\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \bar{a}e^{y_2-y_1} - \bar{b} - \bar{d}e^{y_1} - \bar{p}e^{y_3} \\ \bar{c}e^{y_1-y_2} - \bar{f}e^{y_2} \\ -\bar{g} + \bar{h}e^{y_1} - (\bar{r} | \bar{q})e^{y_3} \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

If system (2) does not have a solution, then naturally

$$QN\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

holds. This proves that the condition (b) of Lemma 1 holds.

Finally we will prove that condition (c) of Lemma 1 holds. To this end, we define $\phi: \mathrm{Dom}L \times [0,1] \to X$ by

$$\varphi(y_1, y_2, y_3, \mu) = \begin{bmatrix} \bar{a}e^{y_2-y_1} - \bar{b} \\ \bar{c}e^{y_2-y_2} - \bar{f}e^{y_2} \\ -\bar{g} + \bar{h}e^{y_1} - (\bar{r} + \bar{q})e^{y_3} \end{bmatrix} + \lambda \begin{bmatrix} \bar{d}e^{y_1} - \bar{p}e^{y_3} \\ 0 \\ 0 \end{bmatrix}$$

where $\mu \in [0,1]$, here $(y_1, y_2, y_3)^{\mathrm{T}} \in \partial\Omega \cap \mathrm{Ker}L = \partial\Omega \cup R^3$, $(y_1, y_2, y_3)^{\mathrm{T}}$ is a constant vector in R^3 with $|y_1| + |y_2| + |y_3| = M$. Next, we can show that when $(y_1, y_2, y_3)^{\mathrm{T}} \in \partial\Omega \cup \mathrm{Ker}L$. $\varphi(y_1, y_2, y_2, \mu) \neq 0$. if the conclusion is not true, constant vector $(y_1, y_2, y_3)^{\mathrm{T}}$ with $|y_1| + y_2| + |y_3| = M$ satisfying $\varphi(y_1, y_2, y_2, \mu) \neq 0$, then from

$$\begin{cases} \bar{a}e^{y_2-y_1} - \bar{b} - \bar{d}e^{y_1} - \bar{p}e^{y_3} = 0 \\ \bar{c}e^{y_1-y_2} - \bar{f}e^{y_2} = 0 \\ -\bar{g} + \bar{h}e^{y_1} - (\bar{r} + \bar{q})e^{y_3} = 0 \end{cases}$$

following the argument of (15), (16), (17) and (22) gives

$$|y_i| < \max\{|\ln A_i|, \rho_i\} \quad (i = 1,2,3).$$

Thus

$$|y_1| + |y_2| + |y_3| < \max\{|\ln A_1|, \rho_i\} + \max\{|\ln A_2|, \rho_2\} + \max\{\ln A_3|, \rho_3\} < M,$$

which contradicts the fact that $|y_1| + |y_2| + |y_3| = M$. Furthermore, taking $J = I$: $\mathrm{Im}Q \to \mathrm{Ker}L$, $(y_1, y_2, y_3)^{\mathrm{T}} \to (y_1, y_2, y_3)^{\mathrm{T}}$ we have

$$\deg(JQN(y_1, y_2, y_3)^{\mathrm{T}}, \Omega \cap \mathrm{Ker}L, (0,0,0)^{\mathrm{T}}) = \deg(\varphi(y_1, y_2, y_3, 0), \Omega \cap \mathrm{Ker}L, (0,0,0)^{\mathrm{T}})$$
$$= \deg\{[\bar{a}e^{y_2 - y_1} - \bar{b}, \bar{c}e^{y_1 - y_2} = \bar{f}e^{y_2}, -\bar{g} + \bar{h}e^{y_1}]^{\mathrm{T}}, \Omega \cap \mathrm{Ker}L, (0,0,0)^{\mathrm{T}}\} = -1.$$

This verifies condition (c) of Lemma 1. By now we know that Ω satisfies all the requirement of Lemma 1 thus system (2) has at least one ω-periodic solution. This completes the proof.

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